PHYSICS 534

ANSWERS

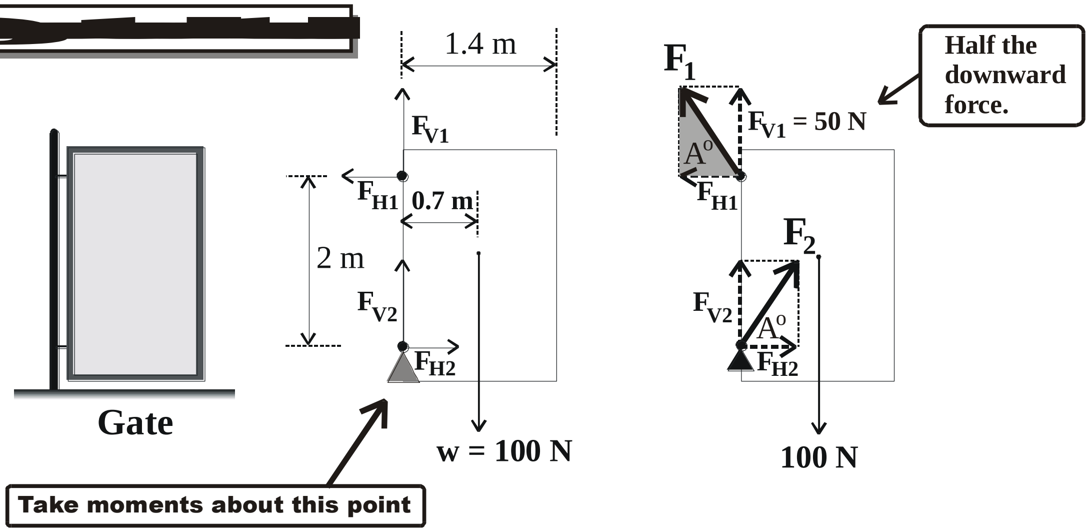
EXERCISE-22 **Equilibrium Review Part-2/ 2**

STARK

Johannes Stark received the Nobel prize for physics in 1919 for his work on atomic spectral lines.

1. A uniform gate has a mass of 10 kg. The gate is 1.4 m wide with hinges 2 m apart. Using the vector diagrams below, calculate the forces (F_1 and F_2) exerted by each hinge. [61 N at 55^o N of W] [61 N at 55^o N of E]

[redacted]



Note that the component forces F_{V1} and F_{V2} each carry half the weight or $F_{V1} = F_{V2} = 50$ N

To calculate F_{H1} (and F_{H2}) we take moments about the bottom hinge:

Relative to the chosen fulcrum, note that the moment of F_{V1}, F_{V2} and F_{H2} is zero.

$$\because\ \Sigma \text{ cwm} = \Sigma \text{ ccwm}$$

$$(100\text{ N})(0.7\text{ m}) = (F_{H1})(2\text{ m})$$

$$\therefore\ F_{H1} = \frac{(100\text{ N})(0.7\text{ m})}{2\text{ m}} = 35\text{ N}$$

To find force F_1 (and F_2), with reference to the shaded triangle:

$$\because\ F_1^2 = F_{H1}^2 + F_{V1}^2$$

$$\therefore\ F_1^2 = (35\text{ N})^2 + (50\text{ N})^2$$

$$F_1 = 61\text{ N}$$

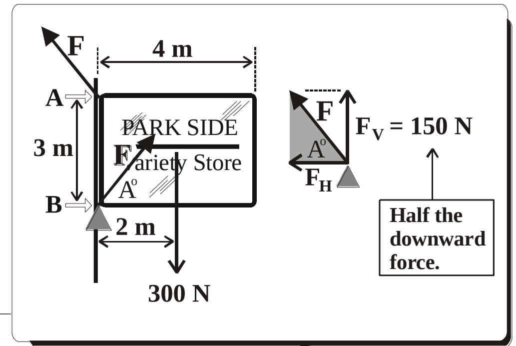
To find the angle (A) we use the Tan function:

$$\text{Tan A} = \frac{50\text{ N}}{35\text{ N}} = 1.4285 \qquad \therefore\ \text{A} = 55^o$$

Answer: $F_1 = 61$ **N [W 55^o N]** and $F_2 = 61$ **[N E 55^o N]**

2. A

metal plates (A and B) at the top and bottom of the sign. The sign measures 4 m wide by 3 m high and has a mass of 30 kg with its center of gravity in the middle of the sign. Find the forces exerted by each metal plate used to support the sign. [250 N at 37^o N of W] [250 N at 37^o N of E]



Taking moments about the botom metal plate:

$\because\ \Sigma \text{cwm} = \Sigma \text{ccwm}$

$\therefore\ (300\ N)(2\ m) = (F_H)(3\ m)$

$$F_H = \frac{(300\ N)(2\ m)}{3\ m} = 200\ N$$

$\because\ F^2 = F_V^2 + F_H^2$ or $F^2 = (150\ N)^2 + (200\ N)^2$ $\therefore\ F = 250\ N$

And, with reference to the shaded triangle:

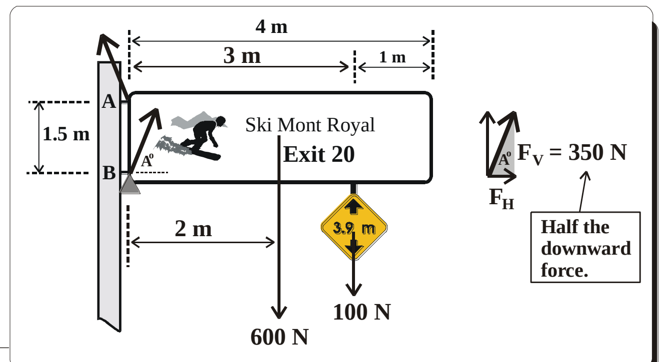
$$\text{Tan } A = \frac{150\ N}{200\ N} = 0.75 \quad \therefore\ A = 36.8^o = 37^o$$

Answer: 250 N [W 37° N] and 250 N [E 37° N]

3. A highway sign is attached to a pole by means of two braces (A and B) which are 1.5 m apart. The length of the sign is 4 m. It is made of uniform construction with a mass of 60 kg.

Attached to the sign, 1 m from the far end, is a smaller sign having a mass of 10 kg. Calculate the force exerted by each brace.

[1060 N 19^o N of W] [1060 N 19^o N of E]



Taking moments about the botom metal plate:

$\because\ \Sigma \text{cwm} = \Sigma \text{ccwm}$

$\therefore\ (600\ N)(2\ m) + (100\ N)(3\ m) = F_H(1.5\ m)$

$$F_H = \frac{(600\ N)(2\ m) + (100\ N)(3\ m)}{1.5\ m} = 1000\ N$$

$\because\ F^2 = F_V^2 + F_H^2$ or $F^2 = (350\ N)^2 + (1000\ N)^2$ $\therefore\ F = 1060\ N$

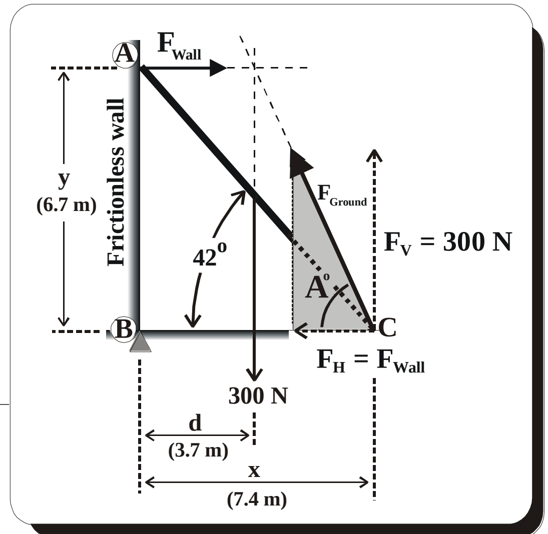
And, with reference to the shaded triangle:

$$\text{Tan } A = \frac{350\ N}{1000\ N} = 0.35 \quad \therefore\ A = 19.2^o = 19^o$$

Answer: 1060 N E 19° N] and 1060 N [W 19° N]

4. A 10 m long ladder is placed against a frictionless wall at an angle of 42^o as illustrated in the diagram on the right. The ladder is uniform and has a mass of 30 kg. Find the force exerted by the wall (F_{Wall}) and the force exerted by the ground (F_{Ground}) which support the ladder.

[166 N East] [343 N 61^o N of W]



Since the wall is frictionless, the only force it exerts is a normal force. That is, a force perpendicular to and out from the wall (F_{Wall}).

Note that the ground force (whose angle is not 42^o) can be resolved into a horizontal component (F_H) and a vertical component (F_V). Moreover, the vertical component force (F_V), being the only upward force (since the wall is frictionless), equals the total downward force. And, the horizontal component of the ground force (F_H) equals the wall force (F_{Wall}). Thus, with reference to the shaded treiangle :

$$F_{Ground}^2 = F_H^2 + F_V^2$$

where $F_H = F_{Wall}$ and $F_V = F_w$ (weight of ladder)

$$\therefore \ F_{Ground}^2 = F_{Wall}^2 + F_w^2$$

Calculation of distances (with reference to ΔABC) :

$$y = \text{height of wall} = (10\text{ m})(\text{Sin } 42^o) = (10\text{ m})(0.6691) = 6.69\text{ m} = 6.7\text{ m}$$

$$x = \text{floor distance} = (10\text{ m})(\text{Cos } 42^o) = (10\text{ m})(0.7431) = 7.43\text{ m} = 7.4\text{ m}$$

$$d = \text{distance of the center of mass of ladder from wall} = 7.4\text{ m}/2 = 3.7\text{ m}$$

Taking moments at the base of the wall :

$$\Sigma \text{ cwm} = \Sigma \text{ ccwm}$$

$$\therefore \quad (F_{Wall})(6.7\text{ m}) + (F_w)(3.7\text{ m}) = (F_V)(7.4\text{ m})$$

(Note that the moment of F_H, relative to the chosen fulcrum, is zero)

But : $F_w = F_V = 300$ N (weight of ladder)

or $\quad (F_{Wall})(6.7\text{ m}) + (300\text{ N})(3.7\text{ m}) = (300\text{ N})(7.4\text{ m})$

or $\quad (F_{Wall})(6.7\text{ m}) = (300\text{ N})(7.4\text{ m}) - (300\text{ N})(3.7\text{ m})$

or $\quad F_{Wall} = \dfrac{(300\text{ N})(7.4\text{ m}) - (300\text{ N})(3.7\text{ m})}{6.7\text{ m}} = 165.7\text{ N} = 166\text{ N}$

With reference to the shaded triangle :

$$F_{Ground}^2 = F_H^2 + F_V^2$$

But $\quad F_H = F_{Wall} = 166$ N and $F_V = F_w = 300$ N

$$\because \quad F_{Ground}^2 = (166\text{ N})^2 + (300\text{ N})^2 \qquad \therefore \ F_{Ground} = 342.8\text{ N} = 343\text{ N}$$

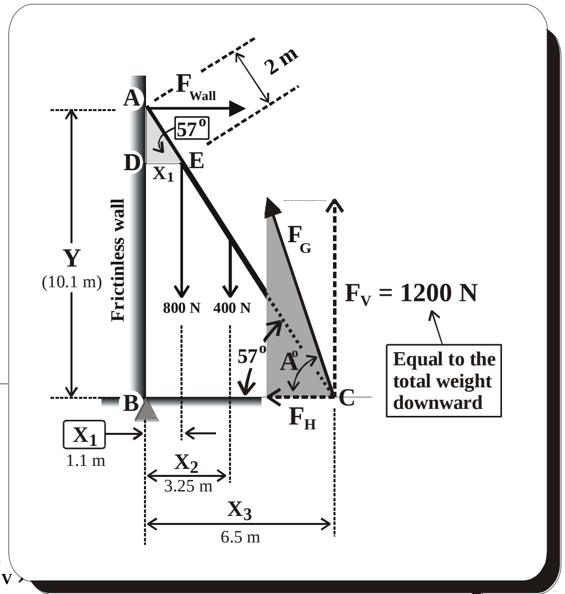
$$\text{Tan A} = \frac{300\text{ N}}{166\text{ N}} = 1.8072 \quad \therefore \ A = 61^o$$

Answer: $F_{Wall} = 300$ N East and $F_{Ground} = 343$ N at 61^o N of W

5. A 12 m long ladder leans against a frictionless wall at an angle of 57° (see the diagram). The ladder is uniform with a mass 40 kg.

Standing on the ladder, 2 m away from the top of the ladder, is an 80 kg man. Determine the forces exerted by the wall (F_{Wall}) and the ground (F_G) supporting the ladder in equilibrium.

[556 N East] [1323 N 65° N of W]



Since the wall is frictionless, the only force it exerts is a normal force. That is, a force perpendicular to and out from the wall (F_{Wall}).

We resolve the ground force (whose angle is not 57°) into a horizontal and a vertical componenet (F_H and F_V).

Calculation of the distances: X_1, X_2, X_3 and Y:

$$\text{From } \Delta \text{ADE}, \; X_1 = (2\text{ m})(\cos 57^\circ) = (2\text{ m})(0.5446) = 1.08\text{ m} = 1.1\text{ m}$$

$$\text{From } \Delta \text{ABC}, \; X_3 = (12\text{ m})(\cos 57^\circ) = (12\text{ m})(0.5446) = 6.53\text{ m} = 6.5\text{ m}$$

$$Y = (12\text{ m})(\sin 57^\circ) = (12\text{ m})(0.8386) = 10.06\text{ m} = 10.1\text{ m}$$

With reference to the bottom (shaded) triangle:

$$F_G^2 = F_H^2 + F_V^2$$

where $F_V = 1200\text{ N}$ **(equal to the total downward force)**

$$F_H = F_{Wall}$$

To find the wall force, F_{Wall}, we take moments at the base of the wall.

$$\Sigma \text{ cwm} = \Sigma \text{ ccwm}$$

$$\therefore \; (F_{Wall})(10.1\text{ m}) + (800\text{ N})(1.1\text{ m}) + (400\text{ N})(3.25\text{ m}) = (1200\text{ N})(6.5\text{ m})$$

$$\text{or} \quad F_{Wall} = \frac{(1200\text{ N})(6.5\text{ m}) - (800\text{ N})(1.1\text{ m}) - (400\text{ N})(3.25\text{ m})}{10.1\text{ m}} = 556.4\text{ N} = 556\text{ N}$$

(Note that the moment of F_H, relative to the chosen fulcrum, is zero)

$$\text{Since} \quad F_G^2 = F_{Walll}^2 + F_V^2$$

$$F_G^2 = (556\text{ N})^2 + (1200\text{ N})^2$$

$$\therefore \; F_G = 1322.5\text{ N} = 1323\text{ N}$$

$$\text{Tan A} = \frac{1200\text{ N}}{556\text{ N}} = 2.1582 \quad \therefore \; A = 65.1^\circ = 65^\circ$$

Answer: $F_{Wall} = 556$ **N East and** $F_G = 1223$ **N at 65° N of W**

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